



Exponents





Exponents

An EXPONENT is the number of times the base number is multiplied by itself.



EXAMPLE: 4^3

4 is the base number. The small, raised number 3 to the right of the base number indicates the number of times the base number is multiplied by itself.

Therefore: $4^3 = 4 \times 4 \times 4 = 64$.

4^3 is read "four to the third power."





Common mistakes



COMMON MISTAKE:

The expression 4^3 does NOT mean 4×3 .





Exponents

Things to remember about exponents

1. Any base without an exponent has a "invisible" exponent of 1


$$8 = 8^1$$





Exponents

Things to remember about exponents

2. Any base with an exponent 0, equals 1



EXAMPLE: $6^0 = 1$



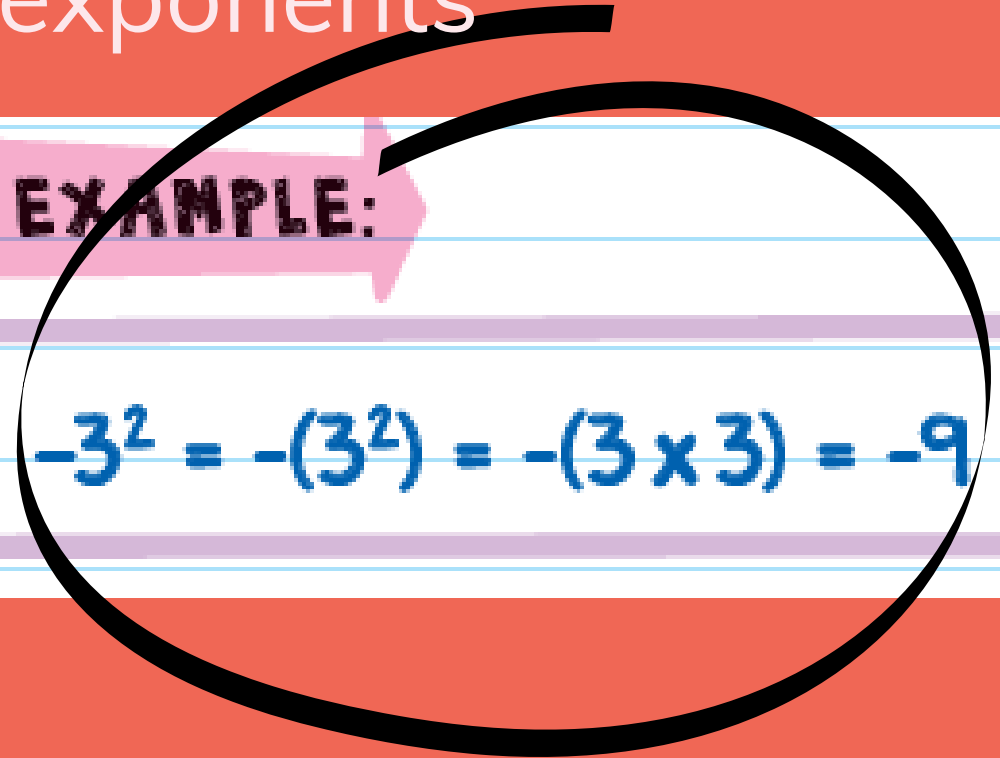
Exponents

Things to remember about exponents

3. Be careful when calculating negative numbers with exponents



EXAMPLE:


$$-3^2 = -(3^2) = -(3 \times 3) = -9 \text{ vs. } (-3)^2 = (-3) \times (-3) = 9$$

Always **LOOK AT WHAT IS NEXT TO THE EXPONENT:**

In the first example, the number **3** is next to the exponent.
So, only the **3** is being raised to the second power.





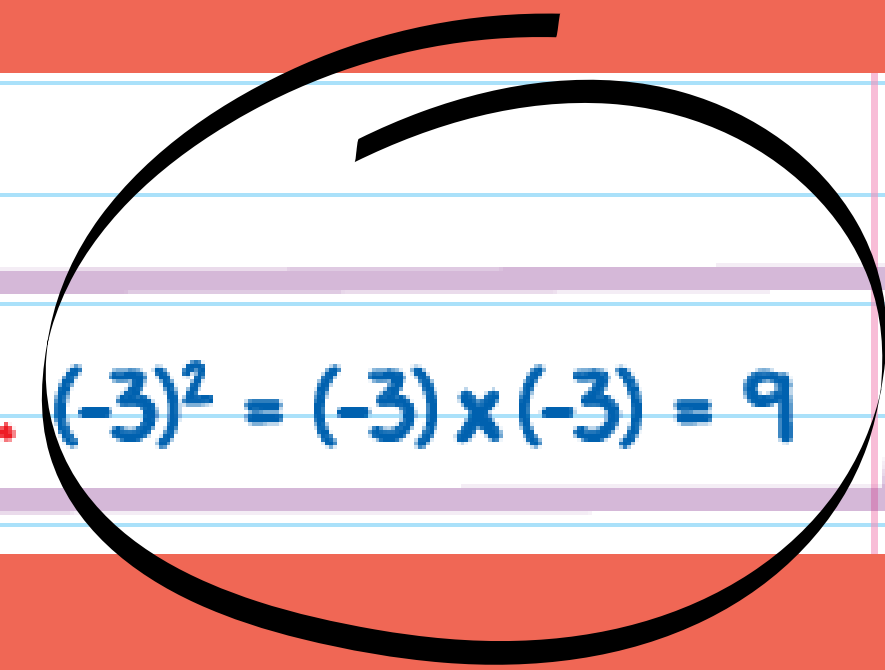
Exponents

Things to remember about exponents

3. Be careful when calculating negative numbers with exponents



EXAMPLE:


$$-3^2 = -(3^2) = -(3 \times 3) = -9 \text{ vs. } (-3)^2 = (-3) \times (-3) = 9$$

In the second example, the parentheses is next to the exponent, so we raise *everything* inside the parentheses to the second power. The **-3** is inside the parentheses and, therefore, **-3** is raised to the second power.





Simplifying

Simplifying expressions with exponents

You can simplify expressions with more than one exponent by combining the exponents-the only requirement is that **the base must be the same**. It looks like this:



$$x^a \cdot x^b = x^{a+b}$$

$$x^a \div x^b = x^{a-b}$$



Simplifying

Examples

When multiplying powers with the same base, write the base once, and then add the exponents!

$$5^8$$

EXAMPLE: $5^2 \cdot 5^6 = 5^{2+6} = 5^8$

If you want to check that this works, try the long way:

$$5^2 \cdot 5^6 = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^8$$

Simplifying

Examples

When dividing powers with the same base, write the base once and subtract the exponents!

EXAMPLE: $7^6 \div 7^2 = 7^{6-2} = 7^4$

If you want to check that this works, try the long way:

$$\frac{7^6}{7^2} = \frac{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7}{7 \cdot 7} = 7^4$$

(We can cancel out two of the 7s on top and both on the bottom because anything divided by itself equals 1.)

$$\frac{7^6}{7^2} = \frac{7 \cdot 7 \cdot 7 \cdot 7 \cdot \cancel{7} \cdot \cancel{7}}{\cancel{7} \cdot \cancel{7}} = 7^4$$



Simplifying

Let's try with variables



EXAMPLE:

$$x^2 \cdot 2y \cdot x^4$$

To simplify, we keep the base (x) and add the exponents $2 + 4$.

$$= x^6 \cdot 2y$$

← CAN ALSO BE WRITTEN AS $2x^6y$





Simplifying

Let's try with variables



EXAMPLE: $3a^9 \div 7a^5$ To simplify $a^9 \div a^5$, we keep the base (a) and subtract the exponents $9-5$.

$$= 3a^4 \div 7$$

DON'T FORGET THAT YOU CAN ALSO
FORMAT THIS QUESTION LIKE A FRACTION
IF IT MAKES THE SOLUTION EASIER TO SEE.


$$\frac{3a^9}{7a^5}$$




Power of a power

When there is an exponent inside parentheses and another outside the parentheses, this is called a **POWER OF A POWER**. A power of a power can be simplified by multiplying the exponents. It looks like this:

$$(v^a)^b = v^{a \cdot b}$$





Power of a power

Examples

EXAMPLE: $(4^2)^3 = 4^{2 \cdot 3} = 4^6$

If you want to check that this works, try the long way:
 $(4^2)^3 = 4^2 \times 4^2 \times 4^2 = 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$

$$[(-7)^2]^3 = (-7)^2 \cdot (-7)^2 \cdot (-7)^2 = (-7)^{2+2+2} = (-7)^6$$





Power of a power

Examples

EXAMPLE:

$$(3x^7y^4)^2 = 3^{1 \cdot 2} \cdot x^{7 \cdot 2} \cdot y^{4 \cdot 2} = 3^2 \cdot x^{14} \cdot y^8 = 9x^{14}y^8$$

(Don't forget: Any base without an exponent has an "invisible" exponent of 1.)





Power of a power



**Mnemonic for “Power of a
Power: Multiply Exponents”:**

Powerful **O**rangutans **P**ropelled
Multiple **E**lephants.



Don't forget





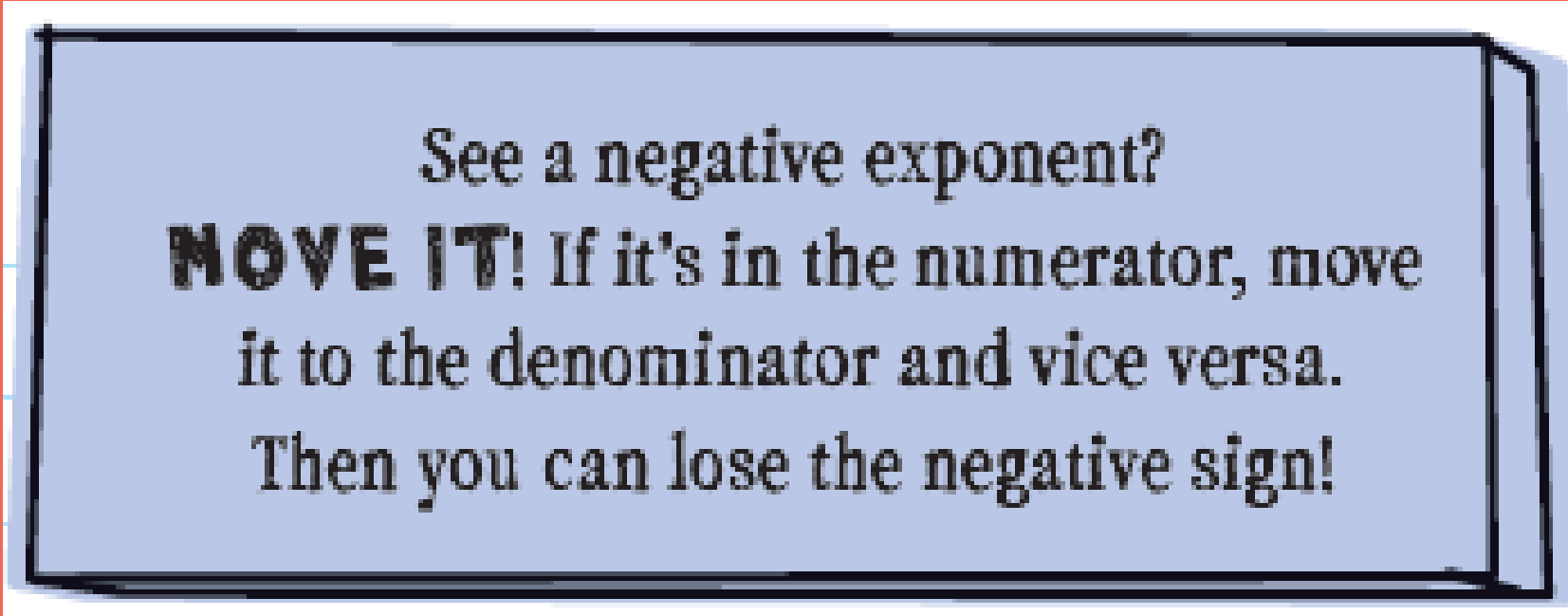
Negative Exponents

You can easily calculate a negative exponent by using reciprocals



A negative exponent in the numerator becomes a positive exponent when moved to the denominator. It looks like this:

$$x^{-m} = \frac{1}{x^m}$$



See a negative exponent?
MOVE IT! If it's in the numerator, move it to the denominator and vice versa. Then you can lose the negative sign!



Negative Exponents

Examples

EXAMPLE: $3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

And the opposite is true: A negative exponent in the denominator becomes a positive exponent when moved to the numerator. It looks like this:

$$\frac{1}{x^{-m}} = x^m$$

EXAMPLE: $\frac{1}{5^{-2}} = 5^2 = 25$

Negative Exponents

Examples

EXAMPLE:

$$\frac{x^5 y^{-3}}{x^{-4} y^4}$$

Turn y^{-3} into y^3 by moving it to the denominator.

Turn x^{-4} into x^4 by moving it to the numerator.

The new expression is

$$\frac{x^5 \cdot x^4}{y^3 \cdot y^4}$$

It simplifies to

$$\frac{x^9}{y^7}$$



Exercises



1. 5^3



2. $14m^0$



3. -2^4



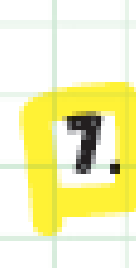
4. $x^9 \cdot x^5$



5. $4x^2 \cdot 2y \cdot -3x^5$



6. $\frac{f^9}{f}$



7. $\frac{-15x^4y^2}{5x^3y^2}$



8. $(10^3)^2$



9. $(8m^3n)^3$



10. $\frac{y^5z^{-2}}{y^2z^6}$

